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Tuning of a Novel I-PD compensator used with Second-Order-Like Processes

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Abstract:

This paper presents a new control compensator from the second generation for use to control second-order-like processes. The process covers a wide range of damping ratio from 0.2 to 6 and damping ratio from 1 to 10 rad/s. The compensator parameters are tuned for optimal characteristics including zero steady-state error, zero maximum percentage overshoot and small settling time. The tuned compensator parameters are reduced to only one set covering the whole range of process parameters. The used compensator tuning technique is compared with a tuning technique based on minimum ITAE standard forms.

Keywords — **Novel I-PD compensator, compensator tuning, second-order-like processes, control system performance**.

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I. INTRODUCTION

Control compensator is a class of controllers used to control various processes with structure and location possibility different than that of controllers. This research work by the author is focused on presenting new generation of control compensators providing better performance for the control system. The start is with a literature survey regarding the use of control compensators in linear control applications.

Lin (1965) in his master thesis determined the minimum amplifier gain and the number of compensator poles and zeros based on the desired specifications. He presented two examples for the application of his suggested compensator design approach [1]. Chen (1988) used the frequency response approach to design a lag-lead compensator to control a robot arm. He compared with a PD controller and came out with a conclusion that the use of the lag-lead compensator reduced the tracking errors considerably [2]. Karunasinghe (2003) presented a simplified lead-lag compensator design to control a XY table as a test bed. His design was able to develop an explicit expression for the compensator for a desired phase margin [3].

Nassirharand and Karimi (2004) presented a closed-form solution for the design of lead-lag compensators having 2/2 transfer function. They developed an expert system for the design of lead-lag compensators based on the definition of the desired transfer function [4]. Horng (2012) used the genetic algorithms to design a lead-lag compensator including the design specifications into the cost function of the genetic algorithm iterations. He claimed that the performance of their design approach of the compensator was good [5]. Singh, Das, Basumatary and Roy (2014) used the root locus and frequency response approach to tune a 1/1 compensator having three parameters. They applied their approach to control processes with single and two simple poles plus an integrator. They investigated also the use of a 2/2 lag-lead compensator. The maximum percentage overshoot associated with the tuned compensators was as large as 11.9, 16.6, 17.2, 20 and 30.4 % depending on the type of compensator and the controlled process [6].

Hassaan (2014) originated a new expression in control engineering called '*second generation of control compensators*'. The objective of this expression was to look for new feedback, feedforward ar feedback/feedforward compensators capable of producing better performance of control systems incorporating processes having bad or unstable dynamics. He introduced a novel feedback PD compensator to control a third-order process [7], a novel feedforward 2/2 second order compensator to control a very slow second order process [8], a novel Notch compensator to control a highly oscillating second-order process [9], a novel feedback PD compensator to control underdamped second-order processors [10], a feedforward lag-lead second-order compensator to control a highly oscillating second-order process [11], a novel Sallen-Key compensator to control a highly oscillating second-order process [12] and a novel feedback first-order compensator to control a highly oscillating second-order process [13].

Hassaan (2015) continued his research to introduce more compensators to the second generation of control compensators. He introduced a second-order compensator to control a highly oscillating second-order process [14], a feedforward third order compensator to control underdamped second order processes [15]. He introduced a new tuning technique for the old first-order compensator when used to control a very slow second-order process [16] and a double integrating process [17]. Hassaan (2021) tuned a number of compensators belonging to the first generation of compensators and compared them with some compensators from the second-generation [18]. Hassaan (2022) introduced a novel P-D compensator to control a highly oscillating second-order-like process [19] and to control a dual liquid tank process [20].

II. SECOND-ORDER-LIKE PROCESS

A large class of industrial components and processes have dynamics that can be identified as second order processes. This is why we say 'like' while taking about their dynamics. A second order process has a transfer function, $G_p(s)$ defined by the mathematical model [21]:

$$G_p(s) = \omega_n^2/(s^2+2\zeta\omega_n s+\omega_n^2) \qquad (1)$$

Where:

ω_n = natural frequency of the process

ζ = its natural frequency (rad/s)

Depending on the value of the damping ratio ζ, the second-order process can be classified as underdamped (if $\zeta<1$), critically damped (if $\zeta<1$) and overdamped (if $\zeta>1$). The following range of the second-order-like process is considered for study in this research work:

$$\left.\begin{array}{l} 1 \le \omega_n \le 10 \ \text{rad/s} \\ \text{and} \quad 0.2 \le \zeta \le 6 \end{array}\right\} \qquad (2)$$

III. STRUCTURE OF THE I-PD COMPENSATOR

The proposed structure of the I-PD compensator is shown in Fig.1. An integral element of transfer function K_i/s is set in the feedforward path just after the error detector of the control system incorporating the compensator and the process. A PD element is set in the feedback path going to the error detector having a transfer function $K_{pc}+K_d s$.

The compensator parameters are:

K_{pc}: Proportional gain.

K_i: Integral gain.

K_d: Derivative gain.

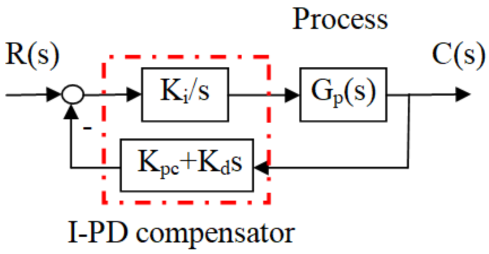


Fig.1 Control system using I-PD compensator.

The proposed compensator can be classified as feedback/feedforward one because of the existence of two elements in both paths.

A. *Transfer Function of the Control System*

Using the block diagram in Fig.1 and the process transfer function in Eq.1, the closed-loop transfer function of the closed-loop control system incorporating the I-PD compensator and the process, M(s) is derived as:

$$M(s)=K_i\,\omega_n^2/\{s^3+2\zeta\omega_n s^2+\omega_n^2(1+K_dK_i)s+K_{pc}K_i\omega_n^2\} \qquad (3)$$

Eq.3 reveals the fact that the closed loop system for step input tracking is a third order one. This means that its dynamics may be unstable, stable with bad dynamics or stable with good dynamics based on the values of the process parameters and the compensator parameters.

B. Stead-state Error

The steady-state error of the control system for a unit step reference input, e_{ss} is derived from Eq.2 and given by:

$$e_{ss} = (K_{pc} - 1)/\,K_{pc} \qquad (4)$$

Eq.4 reveals very important characteristics for the step input tracking control system using the I-PD compensator:

- The steady state error depends on the proportional gain of the I-PD compensator.
- It is possible to achieve zero steady state error if the proportional gain is set to a unit value in all times. That is:

  $$K_{pc} = 1 \qquad (5)$$
- This is the first step in compensator tuning for the optimum performance of the control system incorporating the I-PD compensator.

IV. TUNING THE I-PD COMPENSATOR

- The compensator under study has three gain parameters: K_{pc}, K_i and K_d.
- One of them has already assigned in the above analysis which is the proportional gain K_{pc}.
- Now, we have only two compensator parameters to be tuned for better performance, K_i and K_d.
- The MATLAB optimization toolbox is used for this purpose [22] using its command '*fminunc*'.
- The step time response of the control system is evaluated during optimization application using the '*step*' command of the MATLAB program [23].
- Various performance indices based on the error between the step input and the process step time response, e(t) are tried and the ISE (Integral of square error) is selected to be the used one for compensator tuning. It is defined as:

  $$ISE = \int e(t)^2\,dt \qquad (6)$$
- The ISE of Eq.6 is minimized producing the optimal compensator gains K_i and K_d for process natural frequency and damping ratio in the range given in Eq.2.
- It was found that the optimal values of K_i are almost constant at 3890. That is:

  $$K_i = 3890 \qquad (7)$$
- The optimal values of K_d are very close to each other for all the values of process parameters in Eq.2.
- The mean value of K_d for each process natural frequency and its standard deviation was obtained using the MATLAB commands '*mean*'($K_{d,mean}$) and '*std*' (S_{mean}) respectively [24]. The results are given in Table 1.

TABLE I
MEAN DERIVATIVE GAIN

ω_n (rad/s)	**$K_{d,mean}$**	**S_{mean}**
1	3.48767	0.0925
2	3.46984	0.02178
4	3.48168	0.01016
6	3.45121	0.07737
8	3.48993	0.00568
10	3.4922	0.00451

- An overall mean for the derivative gain, $K_{d,mean,All}$ is obtained using the '*mean*' command and its standard deviation, S_{All} is obtained using the command '*std*'. The results are as follows:

$$K_{d,mean,All} = 3.48755 \text{ and } S_{All} = 0.01569 \qquad (8)$$

- Graphically, the variation of the mean derivative gain $K_{d,mean}$ with the process natural frequency ω_n is shown in Fig.2 with the overall mean value $K_{d,mean,All}$.
- To investigate the effect of considering the mean values of the derivative gain, on the tracking step time response of the control system is obtained for the following process parameters:

$$\omega_n = 1 \text{ rad/s}, \zeta = 6 \quad (9)$$

- The exact derivative gain value for process parameters in Eq.9 is:

$$K_d = 3.41583 \quad (10)$$

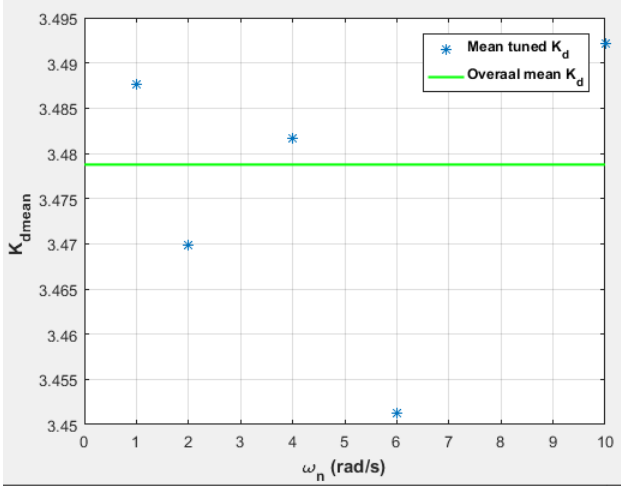


Fig.2 Mean derivative gain of the I-PD compensator.

- The mean derivative gain value for a 1 rad/s natural frequency of the process using Table I is:

$$K_d = 3.48767 \quad (11)$$

- The unit step time response of the control system using Eqs.5, 7, 8, 10 and 11 for the tuned I-PD compensator parameters with exact K_d, mean K_d over damping ratio and overall mean K_d is shown in Fig.3.

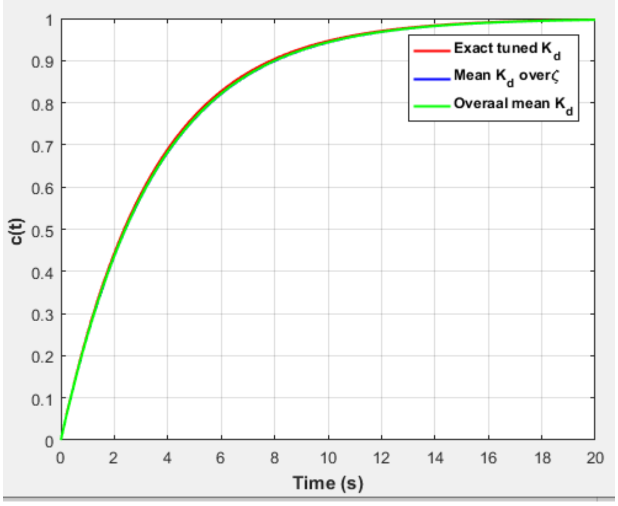


Fig.3 Step time response using I-PD compensator for process with ω_n = 1 rad/s and ζ = 6.

- The three unit step responses are very close to each other, i.e. they are highly correlated. This correlation is measured by the '*correlation coefficient*' obtained in MATLAB using the command '*corrcoef*' [24]. The correlation coefficient for the plots in Fig.3 is:
  0.999957 between time response with exact K_d value and mean K_d value over $\zeta = 6$.
  0.999967 between time response with exact K_d value and overall mean K_d.

V. COMPARISON WITH TUNING USING MINIMUM ITAE STANDARDS FORMS

- To check the efficiency of the tuning approach used in this research work to tune the I-PD compensator used to control second-order-like processes, it was compared with using the minimum ITAE standard forms [25].
- The optimum characteristic equation of a linear control system having a transfer function with a third-order denominator and a zero-order numerator is [25]:

$$s^3+1.75\omega_0 s^2+2.15\,\omega_0^2 s+\omega_0^3 = 0 \quad (12)$$

Where ω_0 is a constant parameter.

- The characteristic equation of the control system incorporating the I-PD compensator using Eq.3 is compared with Eq.12 producing three equations in K_i, K_d and ω_0. Solving them reveals the tuned compensator parameters using the minimum ITAE standard form as:

$$K_i = 1.09278 \quad \text{and} \quad K_d = 9.36387 \quad (13)$$

- The unit step tracking time response of the control system incorporating an I-PD compensator controlling a second order process having the parameters given in Eq.9 is shown in Fig.4 using both tuning techniques presented in this research paper.
- The control system has the compared time-based characteristics:
  - Steady-state error: zero for both tuning techniques.
  - Maximum percentage overshoot: zero for both tuning techniques.
  - Settling time (for ± 2% tolerance band around the steady-state response): 13.645 s for the present tuning technique compared

with 50.0 s for the tuning technique based on the minimum ITAE standard forms.

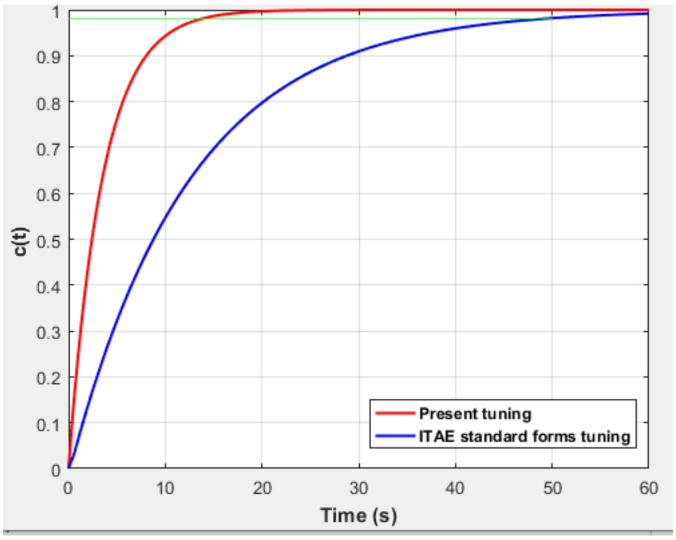
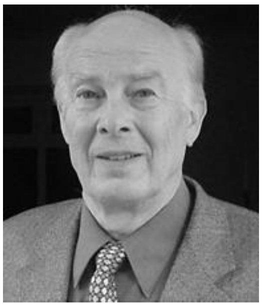


Fig.4 Step time response using I-PD compensator for process with ω_n = 1 rad/s and ζ = 6 with two tuning techniques.

VI. CONCLUSIONS

- This research work investigated the use of a novel I-PD compensator to control second-order-like processes.
- This new compensator belonged to the '*second generation of control compensators*' originated by the author in 2014.
- The compensator had three gain parameters to be tuned for optimum performance of the closed loop control system.
- The compensator was applied to control a set of second-order-like processes having damping ratio from 1 to 10 rad/s and damping ratio from 0.2 to 6.
- The MATLAB optimization toolbox was used to tune the compensator.
- The proportional gain of the compensator was set to a unit value to produce a tracking step time response having zero steady-state error.
- The author followed a tuning approach that could reduce the tuning effort to only one set of compensator parameters to control a second order process in the range assigned for the process-parameters.
- The tuning technique applied was compared with the minimum ITAE standard forms tuning technique.
- The improvement in the tracking step time response was superior using the tuning technique presented by the author.

DEDICATION



Late Professor John Parnaby

- Prof. J. Parnaby was my Ph.D. supervisor in Bradford University, UK during the period from 1974 to 1979.
- He was a professional professor in industrial engineering with academic and field experience.
- I taught 'Advanced Automatic Control' from him and he made me like 'Experimental System Dynamics'.
- Therefore, I have the honour to dedicate this work to him.
- Thanks dear Professor.
- Your student Galal Hassaan from Egypt.

BIOGRAPHY

Galal Ali Hassaan

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.

- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
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- Research on Automatic Control, Mechanical Vibrations, Mechanism Synthesis and History of Mechanical Engineering.
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- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Chief Justice of the International Journal of Computer Techniques.
- Honour Chief Editorial of IJCT.
- Reviewer in some international journals.
- Scholars interested in the authors publications can visit: **http://scholar.cu.edu.eg/galal**

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