RESEARCH ARTICLE

The Summary of Fault Tolerant Hamiltonian Properties of Data Center Network

Shuai Ding*, ZhiyuZhao**
*(College of Computer Information and Engineering, Henan University,
Kaifeng, Henan 475000 ,China
Email:shuai_ding@vip.henu.edu.cn)
** (College of Computer Information and Engineering, Henan University,

Kaifeng, Henan 475000 ,China

Email:104754190928@henu.edu.cn)

--************************----------------------------------

Abstract:

As the infrastructure of cloud computing, the research of data center network has been the focus in recent years.The data center network is undergoing profound changes, and a large number of highperformance network topology has been proposed. The Hamiltonian Properties of the network has a wide range of applications. Then the use of Hamiltonian circles or Hamiltonian paths on the data center network can effectively reduce or avoid deadlock and congestion. At the same tine, since server or link failures are unavoidable in the network, it is meaningful to study the fault-tolerant Hamiltonian properties. In this paper, First of all, the network structure of the cloud computing data center is discussed from two categories: switch-centric networks and server-centric networks. In addition, the network structure with fault-tolerant Hamiltonian properties is studied, and finally the general method of analyzing fault-tolerant Hamiltonian properties based on structure induction is summarized.

Keywords —**Cloud Computing, Data Center Network, Fault Tolerant Hamiltonian Properties**

--************************----------------------------------

I. INTRODUCTION

In the past decade, cloud computing and data center networks have developed rapidly. Cloud computing has been widely used in industry and academia.It provides an innovative way to organize computing resources and plays an increasingly important role in education, scientific research, energy, medical and other related fields [1]. As the infrastructure of cloud computing, the data center network connects a large number of servers through high-speed links and switching equipment. It is the physical carrier for the data center to provide users with information services[2]. The performance of the data center network largely determines the performance of cloud computing, so the performance of cloud computing can be improved by building a data center network with good performance. In recent years, people have studied some network structures of data center, which are mainly divided into two categories: switch-centric and server-centric network structures.

In a switch-centric network structure, the server is only responsible for sending data packets to the network and receiving data packets fromthe network, while the switch looks for communication paths between servers and relays data packets. For example, structures such as Fat-Tree [3] and VL2 [4] fall into this category. On the other hand, in a server-centric network, servers act not only as source or target nodes, but also as relay nodes for each other. Tasks such as routing and addressing are placed on the server, and the switch acts as a link between packets from one side to the other. For example, structures such as DCell [5], BCube[6], FiConn[7] and HCN[8], are all server-centric network topologies. Data center network can be expressed asa simple undirected graph G=(V(G),E(G)),V(G) andE(G)respectively represent vertex set and edge set in graph G, while vertex and edge respectively represent server and link connecting server in data center network. The Hamiltonian properties of the network has important applications in information communication[5]. If the Hamiltonian path or the Hamiltonian circle is used in the multicast routing algorithm of the data center network, it can effectively reduce or avoid deadlock and congestion. At the same time, as the scale of data center networks continues to expand, it is inevitable that servers or links will fail, so it is very meaningful to study the fault-tolerant Hamiltonian properties of data center networks.

This work is organized as follows, Section Ⅱ categorizes the data center network topology.Then, Section Ⅲ analyzes and summarizes the fault-tolerant Hamiltonian properties of the data center network structure and general proof methods. Finally, we make a conclusion in Section Ⅳ.

II. THE CLASSIFICATION OF DATA CENTER NETWORK TOPOLOGIES

A. Switch-centric network topology

Fat-Tree: In order to solve the problems of poor scalability of traditional data center networks, high construction costs, small network bandwidth and single node failure, Mohammad Al-Fares et al. proposed the Fat-Tree network structure. As shown in Fig.1, the Fat-Tree structure uses three layers of ordinary switches for networking, from top to bottom are the core layer, the aggregation layer and the edge layer. It not only adopts a large number of cheap commercial switches to avoid the high cost of expansion. In addition, a fully connected method is adopted at the aggregation layer and the edge layer, and multi-path routing technology is adopted to provide a large bisection bandwidth for server communication.

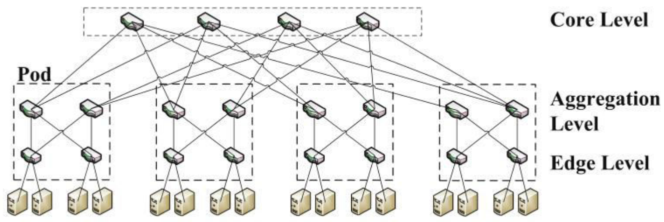

Fig. 1Fat-tree network structure

VL2: Albert Greenberg et al. proposed the VL2 (Virtual Layer 2) data center network structure to improve the scalability of the tree structure and the flexibility of dynamic resource allocation. As shown in Fig.2, the physical topology of the structure is three layers, and the logical topology is two layers. The aggregation layer switches and the core layer switches are completely interconnected to form a complete two-part graph. In addition, the structure has flexible expansion, can support a large number of servers, and can ensure high network bandwidth between servers.

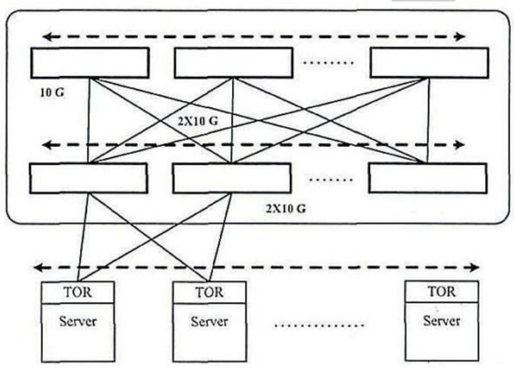

Fig. 2 VL2 network structure

B. *Server-centric network topology*

DCell is a server-centric network structure. At the same time, DCell has excellent scalability and can adapt to the increasing data center network. The DCell network structure is defined in a recursive way, that is, the high-level DCell network structure is fully connected by multiple lower-level DCell network structures. And the basic element is DCell(n, 0), that is DCell$_0$, which has n servers and a mini-switch.Suppose we use $t_{k,n}$ to denote the number of servers of DCell(n,k).For $k \geq 1$, DCell(n, k) is constructed from $t_{k-1,n}+1$ DCell(n, k-1)s.A structure graph of DCell(4,1) is shown in Fig.3.We can find that a DCell(4, 1) is composed of 5 DCells(4, 0).

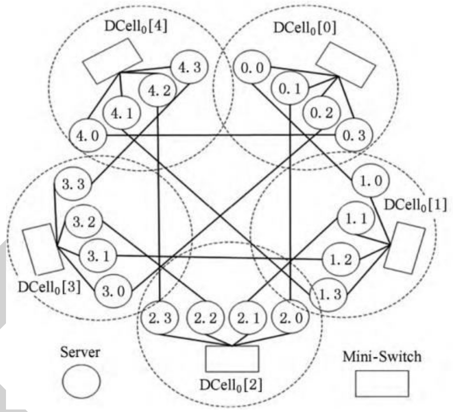

Fig. 3 Network structuregraph of DCell(4, 1)

Similar to DCell(n, k), BCube(n, k) is also a recursively defined network structure. Each server in BCube has k+1 ports, and there are k+1 parallel paths between any two servers. Through this construction method, the BCube network can also provide better scalability and higher network bisection bandwidth. Fig.4 is the network structure graph of BCube(4, 1).

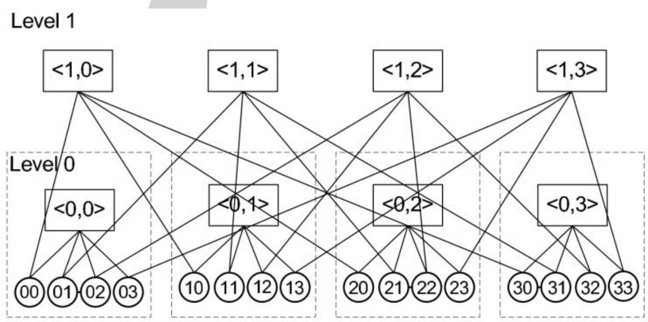

Fig. 4 Network structuregraph of BCube(4, 1)

FiConn and DCellshare the same design principles and are also a recursive structure. However,DCell uses servers equipped with multiple network ports and mini-switches to construct its recursively defined architecture, while FiConn's server node degree is all 2, and it is all composed of dual-port servers, which are interconnected through the available backup ports on each server. The FiConn network is a low-cost structure that avoids the use of expensive switches and does not increase the hardware cost of the server. It also has high scalability and can accommodate hundreds of thousands of servers with low diameter and high bisection bandwidth. Fig 5 is the network structure of FiConn(4, 1).

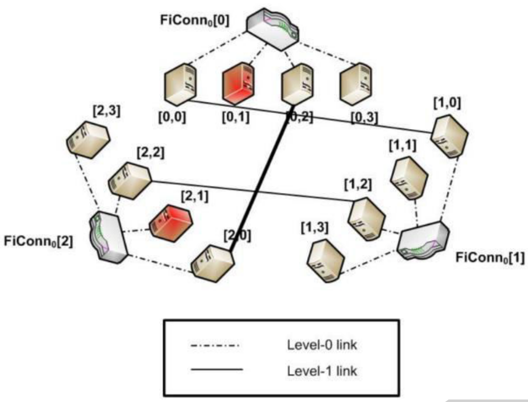

Fig. 5 Network structuregraph of FiConn(4, 1)

The HCN network topology is a recursively defined network structure composed of dual-port servers and n-port commodity switches. In addition, HCN(n, k) is a network structure with excellent performance. And it is an efficient interconnection structure with low equipment cost. Furthermore, it has a high degree of regularity, scalability and symmetry, which is in line with the modular design of the data center. Fig.6 is the network structure graph of HCN(4, 2).

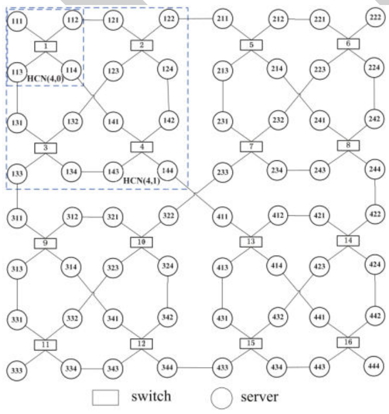

Fig. 6 Network structuregraph of HCN(4, 2)

III. ANALYSIS AND SUMMARY OF FAULT-TOLERANT HAMILTONIAN PROPERTIES BASED ON STRUCTURAL INDUCTION

Generally, various errors will inevitably occur during the operation of large-scale networks. The fault elements in the study of fault tolerance in the network structure are mainly divided into three categories, one is the fault element as a vertex, and the other is the edge. Another category is that in practical applications, the distribution of fault points and edges may be concentrated or localized, that is, fault elements can also be regarded as substructures of the graph, which are called structural faults.

A graph G is Hamiltonian if there exists a Hamiltonian cycle which is a cycle containing all the vertices of the graph G and it is Hamiltonian-connected if there exists a Hamiltonian path which is a path containing all the vertices of the graph G connecting any pair of vertices.

For the fault element set is $F \subseteq V(G) \cup E(G)$ and $|F| \leq f$, graph $G-F$ has a Hamiltonian cycle (respectively, there is a Hamiltonian path between any two distinct vertices), then graph G is called f-fault-tolerant Hamiltonian (respectively, f-fault-tolerant Hamiltonian connected). If the fault element set is $F \subseteq V(G)$ (or $F \subseteq E(G)$) and $|F| \leq f$, graph $G-F$ has a Hamiltonian cycle,then graph G is called f-vertice-fault-tolerantHamiltonian (respectively, f-edge-fault-tolerant Hamiltonian)[9].

A. Fault-tolerant Hamiltonian structures have been proved

DCell is a network structure with excellent performance, and its fault-tolerant Hamiltonian properties have been studied. Wang et al. [10] studied the fault-tolerant Hamiltonian properties of the DCell network, and mainly proved by mathematical induction that for any integer $n \geq 2$ and $k \geq 0$, DCell(n, k) is $(n+k-4)$- fault-tolerant

Hamiltonian connected and $(n+k-3)$- fault-tolerant Hamiltonian.

In the current research on network topology, hypercubes and deformations are often used as network models. Most of the network structure of the data center is based on the hypercube network and its deformation. For example, theBCube structure is based on the generalized hypercube [11] structure. The CamCube [12] network is also based on the data center network topology proposed by a k-ary n-cubes network [13]. Therefore, the study of the fault-tolerant Hamiltonian properties of hypercubes and their deformed structures is of great significance to the fault-tolerant Hamiltonian properties of data center network topology.

Literature [14] proved that for $n \geq 2$, hypercube Q_n is $n-2$ -edge-tolerant Hamiltonian connected. The crossed cube CQ_n is the deformed structure of the hypercube Q_n. Literature [15] gives that for $n \geq 3$, CQ_n is $n-2$ -fault-tolerant Hamiltonian and $n-3$ -fault-tolerant Hamiltonian connected. Then, literature [16] studied for $n \geq 5$, Locally Twisted Cube LTQ_n and Mobius cube MQ_n are $n-2$ -fault-tolerant Hamiltonian connected. Zhang et al. and Ashir et al. studied the fault-tolerant Hamiltonian problem in k-ary n-cube networks in the existence of edge failures [17-18]. Lu et al. studied the fault-tolerant Hamiltonian problem in k-ary n-cube networks based on structural faults, and also proved that Q_n^k is $4(n-2)$ -fault-tolerant Hamiltonian and fault-tolerant Hamiltonian connected [19].

B. Unproven fault-tolerant Hamiltonian structure

So far, there are still a large number of high-bandwidth, high-scalability, and flexible network structures that have not been proven fault-tolerant Hamiltonian properties. For instance, Literature [20] studied the Hamiltonian properties of BCube, but the fault-tolerant Hamiltonian properties of BCube need to be further studied. The fault-tolerant Hamiltonian of high-performance data center network structures such as FiConn and HCN have not been studied yet. To prove and study the fault-tolerant properties of these structures is of great significance to the expansion and fault-tolerant development of data center networks.

C. Summarize the general method of proving fault-tolerant Hamiltonian properties based on Structural induction

Structural induction is a method of proof used in mathematical logic, computer science, graph theory and some other mathematical fields (for example, the proof of Los's theorem). It is a specialized mathematical induction. The proof of structure induction is to prove that the proposition holds for all minimal structures, and if it holds in the basic structure of a structure G, then it must also hold in the whole G.

Let X(n, k) denote DCell(n, k), BCube(n, k), FiConn(n, k), HCN(n, k), etc. X(n, 0) describes the basic element in these network structures, which has n servers and a mini-switch. The data center network structure can be represented as a graph, with vertices representing server nodes in the network, and edges representing links in the network structure. The study of fault-tolerant Hamiltonian properties of data center network structure is actually the study of fault-tolerant Hamiltonian properties of graphs. When we prove fault-tolerant Hamiltonian properties, enumeration or computer search are often used to prove the low level structure of the network. Then, the fault-tolerant Hamiltonian properties of high level network structures are usually proved by structural induction and mathematical analysis. The proof method is mainly divided into the following three steps.

1. X(n, 0) ignores the switch, takes the server as the node, and the link is abstracted as a graph as a connection, and then a complete graph. There is a

Hamiltonian path between any two points in the complete graph, and after removing f (f $\leq n-2$)faulty nodes, any two points still have a Hamiltonian path, so X(n, 0) is *n*-2-fault-tolerant Hamiltonian connected and *n*-3-fault-tolerant Hamiltonian.

2. Since X(n, k) is a recursive structure, X(n, k) is composed of multiple X(n, k-1). According to the structural induction method, we already know that X(n,0) is f-fault-tolerant Hamiltonianconnected in the first step, and then we inductively assume that when k = t-1 ($t \geq 2$), X(n, t-1) is f-fault-tolerant Hamiltonian connected.

3. Through the inductive hypothesis of X(n, t-1) in the second step, and then divide the situation to prove that X(n, t) is f-fault-tolerant Hamiltonian connected. That is, X(n, k) is f-fault-tolerant Hamiltonian connected.

IV. CONCLUSIONS

With the rapid development of cloud computing technology, the field of data center is currently undergoing unprecedented and profound changes. The network topology occupies a major position in the data center. In practice, the vertices and edges of the network may fail, so it is of great practical significance to study the fault-tolerant properties of the network structure.The Hamiltonian property is one of the important indicators to measure the performance of the network structure. In this paper, we mainly analyzes and summarizes the fault-tolerant Hamiltonian properties of the data center network. Firstly, classify the data center network topologies, and then summarize the structure of the data center network that has been proved Hamiltonian and the structure that has not been proven. Finally, we summarize the general proof method of fault-tolerant Hamiltonian properties based on structural induction.

REFERENCES

[1] Shafer J, Rixner S, Cox A L. Datacenter storage architecture for mapreduce applications[C]//ACLD: Workshop on Architectural Concerns in Large Datacenters. 2009, 131.

[2] Wittmann, Art. What Cloud Computing Really Means.[J]. Informationweek, 2010

[3] Alfares M, Loukissas A, Vahdat A, et al. A scalable, commodity data center network architecture[C]. acm special interest group on data communication, 2008, 38(4): 63-74.

[4] Greenberg A , Hamilton J R , Jain N , et al. VL2: A Scalable and Flexible Data Center Network[J]. Communications of the ACM, 2009, 54(3):95-104.

[5] C. Guo, H. Wu, K. Tan, L. Shi, Y. Zhang, and S. Lu, "DCell: A scalable and fault-tolerant network structure for data centers," in Proc. ACM SIGCOMM Conf. Data Commun., Aug. 2008, pp. 75–86.

[6] C. Guo, et al., "BCube: A high performance, server-centric network architecture for modular data centers," in Proc. ACM SIGCOMM Conf. Data Commun., Aug. 2009, pp. 63–74.

[7] Li D , Guo C , Wu H , et al. FiConn: Using Backup Port for Server Interconnection in Data Centers[C]// INFOCOM 2009, IEEE. IEEE, 2009.

[8] Guo D, Chen T, Li D, et al. Expandable and Cost-Effective Network Structures for Data Centers Using Dual-Port Servers[J]. IEEE Transactions on Computers, 2013, 62(7): 1303-1317.

[9] Xiaowen Qin. The fault-tolerant Hamiltonicity of some kinds of networks [D]. Beijing Jiaotong University, 2018.

[10] Wang X, Erickson A, Fan J, et al. Hamiltonian Properties of DCell Networks[J]. The Computer Journal, 2015, 58(11): 2944-2955.

[11] Bhuyan, Agrawal. Generalized Hypercube and Hyperbus Structures for a Computer Network[J]. IEEE Transactions on Computers, 1984, 33(4): 323-333.

[12] Abulibdeh H, Costa P, Rowstron A, et al. Symbiotic routing in future data centers[C]. acm special interest group on data communication, 2010, 40(4): 51-62.

[13] Dally W J. Performance analysis of k-ary n-cube interconnection networks[J]. IEEE Transactions on Computers, 1990, 39(6): 775-785.

[14] Tsai C, Tan J J, Liang T, et al. Fault-tolerant hamiltonian laceability of hypercubes[J]. Information Processing Letters, 2002, 83(6): 301-306.

[15] Huang W T, Chuang Y C, Tan J J, et al. On the fault-tolerant hamiltonicity of fault crossed cubes [J]. IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, 2002, 85(6): 1359-1370.

[16] Zhe Zhang. Fault-Tolerance Research of Locally Twisted Cubes and Möbius cubes [D]. Dalian University, 2017.

[17] Wang S, Zhang S, Yang Y, et al. Hamiltonian path embeddings in conditional faulty k-ary n-cubes[J]. Information Sciences, 2014: 463-488.

[18] Zhang S, Zhang X. Fault-Free Hamiltonian Cycles Passing through Prescribed Edges in k -Ary n-cubes with Faulty Edges[J]. IEEE Transactions on Parallel and Distributed Systems, 2015, 26(2): 434-443.

[19] Lv Y, Lin C, Fan J, et al. Hamiltonian Cycle and Path Embeddings in k-Ary n-Cubes Based on Structure Faults[J]. The Computer Journal, 2016, 60(2): 159-179.

[20] Yinhe Huangfu. Hamiltonian Analysis of Network Structure Based on Basic Elements [D]. Henan University, 2019.